Solutions to JEE Main Home Practice Test - 8 | JEE - 2024

PHYSICS

SECTION-1

1.(B) $P = \dfrac{a - t^2}{bx} \quad \Rightarrow \quad Pbx = a - t^2 \;\Rightarrow\; [Pbx] = [a] = [T^2]$

or $[b] = \dfrac{[T^2]}{[P][x]} = \dfrac{[T^2]}{[ML^{-1}T^{-2}][L]} = [M^{-1}T^4] \;\therefore\; \left[\dfrac{a}{b}\right] = \dfrac{[T^2]}{[M^{-1}T^4]} = [MT^{-2}]$

2.(B) $E_i = 2\,eV,\; E_f = E_2 = -3.4\,eV$

$\therefore$ Energy of photon $E = E_i - E_f = 5.4\,eV$

Work function, $W = \dfrac{12420}{4600} = 2.7\,eV \qquad \therefore\; K_{max} = E - W = 2.7\,eV$

3.(B) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\overrightarrow{BC} - \overrightarrow{BD} = \overrightarrow{AC} - \overrightarrow{AD}$

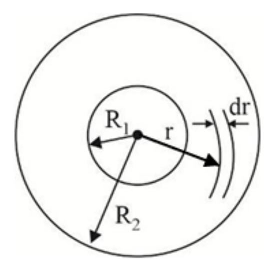
$\overrightarrow{AD} = \overrightarrow{BC}$

$2\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{AC}$

$2\overrightarrow{BC} = 2\hat{i} - \hat{j} + 5\hat{k}$

$\overrightarrow{BC} = \hat{i} - \dfrac{\hat{j}}{2} + \dfrac{5}{2}\hat{k}$

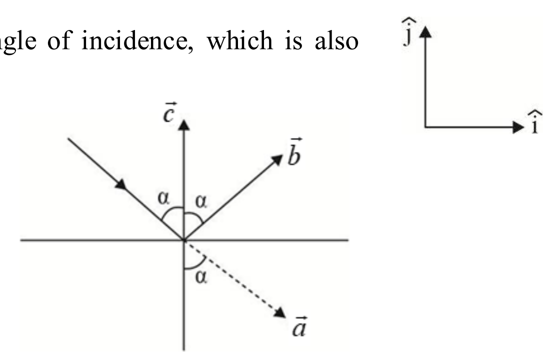
4.(C) Rate of heat flow or heat current will be same throughout the complete section. Therefore, thermal resistance should remain same in between $(R_1 \,\&\, r) \,\&\, (r \,\&\, R_2)$. Let us denote thermal resistance by x.



$\therefore \displaystyle\int_{R_1}^{r} dx = \int_{r}^{R_2} dx; \qquad \int_{R_1}^{r} \frac{dr}{K(4\pi r^2)} = \int_{r}^{R_2} \frac{dr}{K(4\pi r^2)}$

$\therefore \left(\dfrac{1}{R_1} - \dfrac{1}{r}\right) = \left(\dfrac{1}{r} - \dfrac{1}{R_2}\right)$ or $r = \dfrac{2R_1R_2}{R_1 + R_2}$

5.(D) $\vec{a}, \vec{b}$ & $\vec{c}$ are shown in figure. Let α be the angle of incidence, which is also equal to the angle of reflection



$\vec{a} = \sin\alpha\,\hat{i} - \cos\alpha\,\hat{j}$

$\vec{b} = \sin\alpha\,\hat{i} + \cos\alpha\,\hat{j}$

$\therefore\; \vec{b} - \vec{a} = 2\cos\alpha\,\hat{j}$ or $\vec{b} = \vec{a} + 2\cos\alpha\,\hat{j}$

Now, $\vec{a}\cdot\vec{c} = (1)(1)\cos(180° - \alpha) = -\cos\alpha$

$\therefore\; \vec{b} = \vec{a} - 2(\vec{a}\cdot\vec{c})\,\hat{j}$ or $\vec{b} = \vec{a} - 2(\vec{a}\cdot\vec{c})\,\vec{c}$

6.(C) $g = \dfrac{GM}{R^2} = \dfrac{G\left(\frac{4}{3}\rho R^3\right)}{R^2} = \dfrac{4}{3}\rho GR$ i.e., $g \propto \rho R$

R is increased by a factor of 2 i.e., to keep the value of g to be constant the value of ρ has to be changed by a factor of $\dfrac{1}{2}$

7.(A) Let T be the temperature of the mixture. Then $U = U_1 + U_2$

or $\dfrac{f}{2}(n_1 + n_2)RT = \dfrac{f}{2}(n_1)RT_0 + \dfrac{f}{2}(n_2)(R)(2T_0)$

or $(2+4)T = 2T_0 + 8T_0$ $(n_1 = 2,\ n_2 = 4)$ or $T = \dfrac{5}{3}T_0$

8.(A) In each collision A and B will interchange their velocities. So velocity of B after colliding with C.

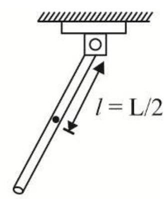
$$v_B = \left(\frac{m-4m}{m+4m}\right)(+v) + \left(\frac{2\times 4m}{m+4m}\right)(0) = -0.6v$$

9.(C) $6\Omega, 3\Omega, 8\Omega, 4\Omega$ & 10Ω resistors make a balanced wheat-stone bridge. Therefore 10Ω resistance can be ignored.

$$R_{AB} = 7 + \frac{9\times 12}{9+12} = \frac{85}{7}\Omega$$

10.(C) $T_1 = 2\pi\sqrt{\dfrac{L}{g}}$; $T_2 = 2\pi\sqrt{\dfrac{I}{Mgl}}$

$$= 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}}; \qquad \frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$$



(physical pendulum)

11.(B) $\Delta l = \dfrac{FL}{AY}$

$$\frac{\Delta l_S}{\Delta l_B} = \left(\frac{F_S}{F_B}\right)\left(\frac{L_S}{L_B}\right)\left(\frac{A_B}{A_S}\right)\left(\frac{Y_B}{Y_S}\right) = \left(\frac{3M}{2M}\right)(a)\left(\frac{1}{b^2}\right)\left(\frac{1}{c}\right) = \frac{3a}{2b^2c}$$

12.(D) At $x_1 = \dfrac{\pi}{3k}$ and $x_2 = \dfrac{3\pi}{2k}$, $\sin kx_1$ or $\sin kx_2$ is not zero.

Therefore, neither of x_1 or x_2 is a node.

$$\Delta x = x_2 - x_1 = \left(\frac{3}{2} - \frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$$

Since, $\dfrac{2\pi}{k} > \Delta x > \dfrac{\pi}{k}$ or $\lambda > \Delta x > \dfrac{\lambda}{2}$; $\left(k = \dfrac{2\pi}{\lambda}\right)$

Therefore, $\phi_1 = \pi$ and $\phi_2 = k.\Delta x = \dfrac{7\pi}{6}$

$\therefore$ $\dfrac{\phi_1}{\phi_2} = \dfrac{6}{7}.$

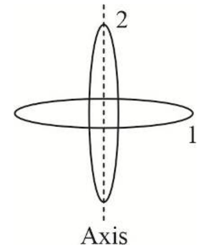
13.(C) $r = \dfrac{\sqrt{2Km}}{Bq} \Rightarrow r \propto \sqrt{\dfrac{K}{B^2}}$

14.(C) $I_1 = mr^2$

$I_2 =$ moment of inertia about one of the diameters

$= \dfrac{1}{2}mr^2$

$\therefore \; I = I_1 + I_2 \qquad = \dfrac{3}{2}mr^2$



15.(D) Magnetic field perpendicular to the plane of the circular loop (in $\hat{k}$ direction) is constant.

16.(C) $\hat{c} = \hat{E} \times \hat{B}$

$\hat{E} = \hat{k}$ and $\hat{B} = \dfrac{2\hat{i} - 2\hat{j}}{\sqrt{4+4}} = \dfrac{1}{\sqrt{2}}\hat{i} - \dfrac{1}{\sqrt{2}}\hat{j}$

Then $\hat{c} = \hat{k} \times \left(\dfrac{\hat{i}}{\sqrt{2}} - \dfrac{\hat{j}}{\sqrt{2}} \right) = \dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{i}}{\sqrt{2}}$

17.(C) $\mathrm{dB} = 10 \log_{10} \left[\dfrac{I}{I_0} \right]$

Where $I_0 = 10^{-12} \text{ Wm}^{-2}$

Since, $40 = 10 \log_{10} \left[\dfrac{I_1}{I_0} \right]$ Or $\dfrac{I_1}{I_0} = 10^4$

Also, $20 = 10 \log_{10} \left[\dfrac{I_2}{I_0} \right]$ or $\dfrac{I_2}{I_0} = 10^2$

or $\dfrac{I_1}{I_2} = 10^2 = \dfrac{r_2^2}{r_1^2}$ or $r_2^2 = 100\, r_1^2$ or $r_2 = 10\,\text{m}$.

18.(A) $|\varepsilon| = \dfrac{d\phi}{dt} = \dfrac{\Delta B}{\Delta t} A \cos 45^\circ = \dfrac{0.1}{0.7} \times 0.1 \times 0.1 \times \dfrac{1}{\sqrt{2}} V$

$i(mA) = \dfrac{0.01}{7\sqrt{2}} \times 10^3 = \dfrac{10}{9.8} = 1\, mA.$

19.(B) Mass of one atom of U^{235} = 235.121420 amu

Mass of one neutron = 1.008665 amu

Sum of the masses of U^{235} and neutron

= 236.130085 = 236.130 amu

Mass of one atom of U^{236}

= 236.123050 amu $\simeq$ 236.123 amu

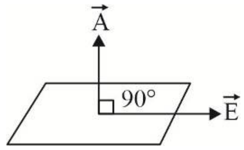
Increase in mass when one neutron is removed from U^{236}

Δm = 236.130 – 236.123 = 0.007 amu

$\therefore$ Energy required to remove one neutron

= 0.007 × 931 MeV = 6.517 MeV $\simeq$ 6.5 MeV

20.(D) Since $\phi = \vec{E} \cdot \vec{A} = EA\cos\theta$



$\theta = 90°$ $\therefore\ \phi = 0$

So, (D) is incorrect

SECTION - 2

1.(20) $Eq = qvB \quad \Rightarrow v = \dfrac{E}{B}$

$$R = \frac{mV}{qB} = \frac{mE}{qB^2} \quad \Rightarrow \quad \frac{q}{m} = \frac{E}{RB^2} = \frac{4\times10^3}{8\times10^{-2}\left(5\times10^{-4}\right)^2}$$

$$= \frac{10^{13}}{50} = 20\times10^{10}\ C/kg$$

2.(2) $\gamma = 1.5\,; \quad p_1 v_1^\gamma = p_2 v_2^\gamma$

$(200)(1200)^{1.5} = p_2(300)^{1.5}$

$p_2 = 200[4]^{3/2} = 1600\,kPa\,; \quad \alpha = 2$

3.(4) $Q_{initial} = CV = 200\times10^{-6}\times200 = 4\times10^{-2}C$

$Q_{final} = KCV = 8\times10^{-2}C$

$\Delta Q = 4\times10^{-2}C$

4.(6) $\dfrac{8D\lambda}{d} = 2.4cm = 2.4\times10^{-2}m$

$\dfrac{8\times1.5\times\lambda}{0.3\times10^{-3}} = 2.4\times10^{-2}$

$\lambda = \dfrac{0.3}{5}\times10^{-5} = 0.6\times10^{-6} = 6\times10^{-7}m$

5.(65) In magnetic field the speed of the charge particle does not change as magnetic force is always perpendicular to velocity vector

$\Rightarrow$ distance = speed × time = 10 × 6.5 = 65 m

6.(52) $9MSD = 10VSD$

$9\times1mm = 10VSD$

$\therefore\ 1VSD = 0.9\,mm$

$LC = 1MSD - 1VSD = 0.1\,mm$

Reading $= MSR + VSR\times LC$

$10 + 8\times0.1 = 10.8\,mm$

Actual reading $= 10.8 - 0.4 = 10.4\,mm$

radius $= \dfrac{d}{2} = \dfrac{10.4}{2} = 5.2\,mm = 52\times10^{-2}\,cm$

7.(5) $y = mx + C$

$v^2 = \frac{20}{10}x + 20$; $\quad v^2 = 2x + 20$

$2v\frac{dv}{dx} = 2 \quad \therefore \; a = v\frac{dv}{dx} = 1 \quad \alpha = 5$

8.(110) $X_L = 2\pi fL$

f is very large

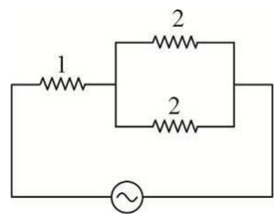
$\therefore \; X_L$ is very large hence open circuit

$X_C = \frac{1}{2\pi fC}$

f is very large

$\therefore \; X_C$ is very small, hence short circuit

Final circuit



$Z_{eq} = 1 + \frac{2\times 2}{2+2} = 2$; $\quad i = \frac{220V}{2\Omega} = 110A$

9.(250) $\frac{\Delta M}{M} = \frac{\Delta \mu}{\mu} = \frac{250}{500} = \frac{1}{2}$

$\frac{1}{2} = \frac{x}{499} \Rightarrow x = 250$

10.(36) Heat produced in a wire of resistance R in time t is

$$H = I^2 Rt$$

Heat required to raise the temperature of a wire by ΔT is

$Q = ms\Delta T \qquad$ Now, $\quad Q = H$

Or $\quad ms\Delta T = I^2 Rt \qquad$ or $\qquad \Delta T \propto I^2$

$\therefore \quad \frac{\Delta T_2}{\Delta T_1} = \left(\frac{I_2}{I_1}\right)^2 = \left(\frac{3I_1}{I_1}\right)^2 = 9$

$\therefore \quad \Delta T_2 = 9\Delta T_1 = 9 \times 4°\text{C} = 36°\text{C}$

CHEMISTRY

SECTION - 1

1.(C) $\Delta G^\circ = -RT \ln K$

$\ln K = \frac{-\Delta G^\circ}{RT}$

$K = e^{-\Delta G^\circ / RT}$

2.(A) Conductance = 1/Resistance

unit $= \frac{1}{\Omega}$ or S ; $\lambda_{eq} = \frac{\kappa \times 1000}{N}$

Unit $\Rightarrow$ Sm^2eq^{-1}

Cell constant $= \frac{\ell}{A}$

Unit $= m^{-1}$

Electrochemical equivalent, $z = \frac{g}{Q}$

Unit $= gA^{-1}s^{-1}$

3.(C) Due to large difference in size of bigger halogens and hydrogen they are less stable than smaller hydrogen halide.

$\therefore$ Stability order : HF > HCl > HBr > HI

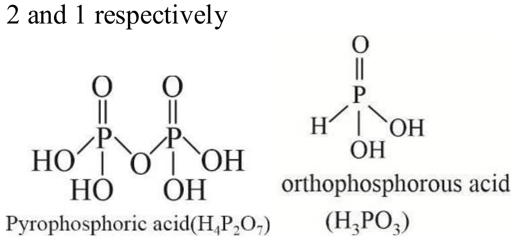
Hence, option (C) is correct

4.(C) I-Q, II-P, III-S, IV-R

5.(D) $K_4\left[Fe\left(CN_6\right)\right]$ in Potassium ferrocyanide Fe has no unpaired electron hence diamagnetic.

$K_3\left[Fe\left(CN_6\right)\right]$ in potassium ferricyanide Fe has unpaired electron hence para magnetic.

6.(C) 2 and 1 respectively



7.(A) Cerium, ${}_{58}Ce : [Xe]4f^1 5d^1 6s^2$

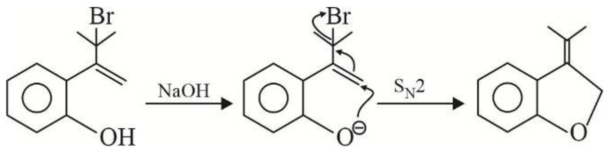
Its most stable oxidation state is +3 but +4 also exists

8.(B) ClO_4^- and MnO_4^- will not show disproportionation because Cl and Mn are in there maximum oxidation state.

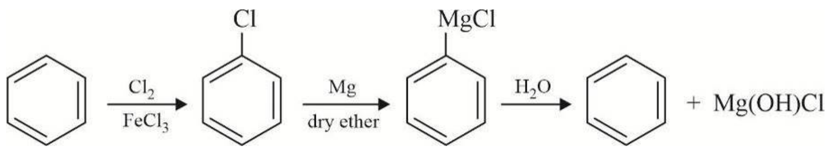
9.(A) $V^{2+} = [Ar]4s^2 3d^3 = t_{2g}^3 e_g^0$

$\mu = \sqrt{n(n+2)} = \sqrt{3(5)} = \sqrt{15} = 3.9$ B.M

10.(A) Conceptual

11.(B) 

12.(A) Stability order: (Anti > Gauche > Partial eclipsed > Eclipsed.)

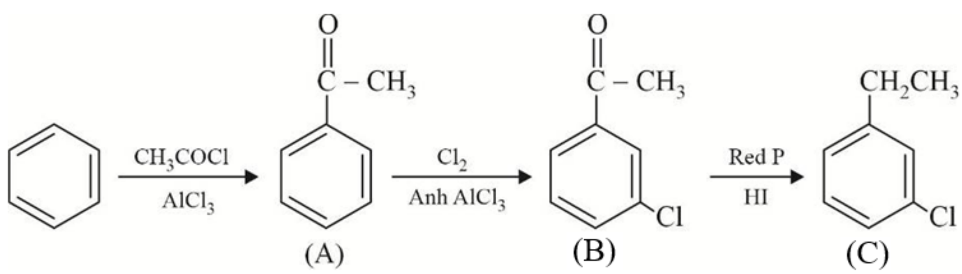
13.(D) 

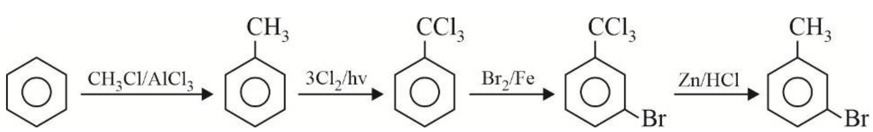
14.(C)

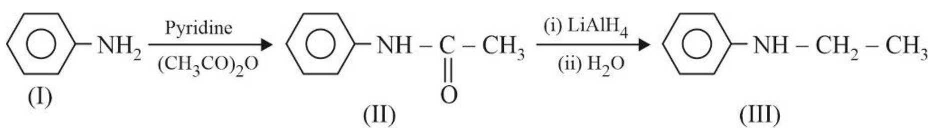
$$HOH_2C-\underset{CH_3}{\overset{CH_3}{C}}-\overset{O}{\overset{\|}{C}}-H \xrightarrow[(ii)\ \Delta]{(i)\ NaOH} HOCH_2-\underset{CH_3}{\overset{CH_3}{C}}-CH_2OH + HOCH_2-\underset{CH_3}{\overset{CH_3}{C}}-COONa$$

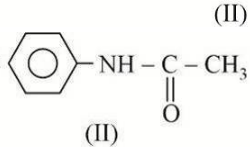
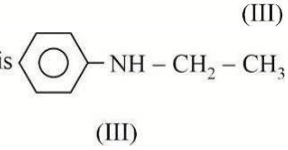
$$\downarrow H_3O^+$$

$$HOCH_2-\underset{CH_3}{\overset{CH_3}{C}}-CH_2OH + HOCH_2-\underset{CH_3}{\overset{CH_3}{C}}-COOH$$

15.(A) 

16.(C) 

17.(A) 

Product (II) is  and (III) is 

18.(A) Conceptual

19.(D) Keratin is a fibrous protein

20.(A) Fact based

SECTION - 2

1.(1) $Na_2O + H_2O \longrightarrow 2NaOH$

$\frac{31}{62}$ moles $\quad = \frac{1}{2}$ moles

Moles of NaOH $= 2\times$ moles of $Na_2O = 2\times\frac{1}{2} = 1$

$[NaOH] = \frac{1}{1} = 1M$

2.(4) KO_2, NO_2, ClO_2, NO are paramagnetic.

3.(0) $[Ar]3d^6 4s^1$

Outermost electron is in 4s sub-shell; $\quad l, m = 0$

4.(1) Molecular orbital configuration of O_2^- is

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, [\pi 2p_x^2 = \pi 2p_y^2], [\pi^* 2p_x^2 = \pi^* 2p_y^1]$

5.(30) $3S \rightleftharpoons S_3$

$1 \qquad 0$

$1-\alpha \qquad \frac{\alpha}{3} \Rightarrow i = 1 - \frac{2\alpha}{3}$

$0.1\left(1-\frac{2\alpha}{3}\right) = 0.08 \Rightarrow \alpha = 0.3.$ Hence 30% trimerization

6.(7)

	HCl +	NaOH $\longrightarrow$	NaCl + H_2O
Initially	50 m moles	50 m moles	–
Finally	0 m mole	0 m mole	50 m mole

pH = 7(neutral solution)

7.(57) $\log_{10} k = 100 - \frac{3\times10^3}{T}$

$\log k = \log A - \frac{E_a}{2.303RT}$

Comparing ; $\quad \frac{Ea}{2.303RT} = \frac{3\times10^3}{T}$

$E_a = 3\times10^3 \times 2.303\times8.314; \quad E_a = 57.44\ kJmol^{-1} \quad \approx 57$

8.(0) Primary valency (x) = 3

Secondary valency (y) = 6

9.(3)

$$3CH_3CH = CH_2 + BH_3 \rightarrow B(CH_2CH_2CH_2)_3$$
$$\downarrow H_2O_2 / OH^-$$
$$3CH_3CH_2CH_2OH + H_3BO_3$$

10.(9) The reaction in dumas method is

$C_xH_yN_z + (2x + y/2)CuO \longrightarrow xCO_2 + y/2H_2O + z/2N_2 + (2x + y/2)Cu$

Comparing; $\quad y = 9$

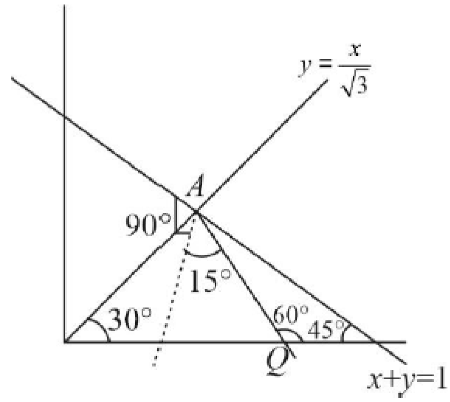
MATHEMATICS

SECTION-1

1.(D) Equation of line $y = mx$

$y = (\tan 30)x$; $y = \dfrac{x}{\sqrt{3}}$

$x + y = 1$ … (i)



For $A \to x + \dfrac{x}{\sqrt{3}} = 1$

$x = \dfrac{\sqrt{3}}{1+\sqrt{3}}$; $y = \dfrac{1}{1+\sqrt{3}}$

$A\left(\dfrac{\sqrt{3}}{1+\sqrt{3}}, \dfrac{1}{1+\sqrt{3}}\right)$

For line AQ, $y - \dfrac{1}{1+\sqrt{3}} = \sqrt{3}\left(x - \dfrac{\sqrt{3}}{1+\sqrt{3}}\right)$

At Q, $-\dfrac{1}{1+\sqrt{3}} = \sqrt{3}\left(x - \dfrac{\sqrt{3}}{1+\sqrt{3}}\right)$; $x = -\dfrac{1}{\sqrt{3}(1+\sqrt{3})} + \dfrac{\sqrt{3}}{1+\sqrt{3}}$

$= \dfrac{1}{1+\sqrt{3}}\left(-\dfrac{1}{\sqrt{3}} + \sqrt{3}\right) = \dfrac{2}{\sqrt{3}(1+\sqrt{3})} = \dfrac{\sqrt{3}-1}{\sqrt{3}}$ $= \dfrac{(1+\sqrt{3})(\sqrt{3}-1)}{\sqrt{3}(1+\sqrt{3})} = \dfrac{2}{3+\sqrt{3}}$

2.(C) $\bar{x} = \bar{b} - \bar{a} \times \bar{x}$

Taking dot product with $\bar{a}$ both side

$\bar{a}\cdot\bar{x} = \bar{a}\cdot\bar{b} - \bar{a}\cdot(\bar{a}\times\bar{x}) \Rightarrow \bar{a}\cdot\bar{x} = 0 - 0 = 0$

$|\bar{b}|^2 = |\bar{x} + \bar{a}\times\bar{x}|^2 = (\bar{x} + \bar{a}\times\bar{x})\cdot(\bar{x} + \bar{a}\times\bar{x})$

$\Rightarrow |\bar{b}|^2 = |\bar{x}|^2 + (\bar{a}\times\bar{x})^2 + [\bar{x}\ \bar{a}\ \bar{x}]\times[\bar{a}\ \bar{x}\ \bar{x}]$

$\Rightarrow |\bar{b}|^2 = |\bar{x}|^2 + |\bar{a}|^2|\bar{x}|^2\sin^2 90° + 0$ $\Rightarrow 1 = |\bar{x}|^2 + |\bar{x}|^2 \Rightarrow |\bar{x}| = \dfrac{1}{\sqrt{2}}$

3.(B) $-1 \le \dfrac{6-3x}{4} \le 1$ & $\dfrac{x-1}{2} \in (-\infty, -1] \cup [1, \infty)$

$\Rightarrow x \in \left[\dfrac{2}{3}, \dfrac{10}{3}\right]$ & $x \in (-\infty, -1] \cup [3, \infty)$ $\Rightarrow x \in \left[3, \dfrac{10}{3}\right]$

4.(D) Choose three elements y_1, y_2, y_3 from 1 to 7 in ${}^7C_3$ ways

So that $f(1) = y_1, f(2) = y_2, f(3) = y_3$ & $y_1 < y_2 < y_3$

Total possible increasing function $= {}^{10}C_4$

$P(E) = \dfrac{{}^7C_3}{{}^{10}C_4} = \dfrac{1}{6}$

5.(C) $f(x+2) = 2f(x) - f(x+1)$

Put $x = 0 \Rightarrow f(2) = 2f(0) - f(1) = 2(2) - 3 = 1$

Put $x = 1 \Rightarrow f(3) = 2f(1) - f(2) = 2(3) - 1 = 5$

Put $x = 2 \Rightarrow f(4) = 2f(2) - f(3) = 2(1) - 5 = -3$

Put $x = 3 \Rightarrow f(5) = 2f(3) - f(4) = 2(5) - (-3) = 13$

6.(B) Consider plane passing through line of intersection of given planes and parallel to z axis

$(x + y + 2z - 3) + \lambda(2x + 3y + 4z - 4) = 0$

$\overline{x} = (2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda + 2)\hat{k}$; $\overline{x} \cdot \hat{k} = 0 \Rightarrow 4\lambda + 2 = 0 \Rightarrow \lambda = -\frac{1}{2}$

$\Rightarrow 2(x + y + 2z - 3) - (2x + 3y + 4z - 4) = 0 \Rightarrow y + 2 = 0$

Perpendicular form $(0,0,0)$ to plane $y + 2 = 0$ is $\left|\frac{0+2}{\sqrt{1}}\right| = 2$

7.(D) Circle $x^2 + y^2 - 2x = 0$ passing $A(\alpha, 0)$ & $B(1, \beta)$

$\Rightarrow \quad \alpha^2 - 2\alpha = 0$ & $\beta^2 - 1 = 0 \because m_{AB} \neq -1$

$ax + by = 0$ pass $A(0, 0)$ & $B(1, 1) \quad \Rightarrow \quad AB$ is $x - y = 0$

Centre of circle whose diameter is AB is $C\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius $= \frac{1}{\sqrt{2}}$

Image of C w.r.t. line $x + y + 2 = 0$ is $C'\left(\frac{-5}{2}, \frac{-5}{2}\right) \quad \Rightarrow \quad x^2 + y^2 + 5x + 5y = 12 = 0$

8.(C) $\sum_{r=1}^{n} \frac{1}{(r+2)\lfloor r} = \sum_{r=1}^{n} \frac{r+1}{\lfloor r+2} = \sum_{r=1}^{n} \frac{r+2-1}{\lfloor r+2} = \sum_{r=1}^{n} \left(\frac{1}{\lfloor r+1} - \frac{1}{\lfloor r+2}\right)$

$a = \lim_{n\to\infty}\left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3}\right) + \left(\frac{1}{\lfloor 3} - \frac{1}{\lfloor 4}\right) + ... + \left(\frac{1}{\lfloor n+1} - \frac{1}{\lfloor n+2}\right) = \frac{1}{2}$

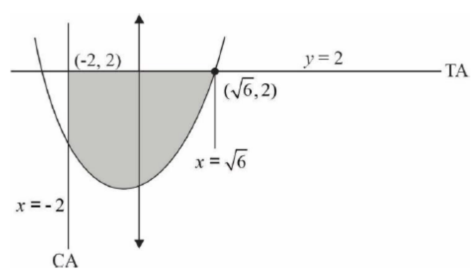
$b = \lim_{x\to 0} e^x \left(\frac{e^{\sin x - x} - 1}{\sin x - x}\right) = e^0 \cdot 1 = 1 \quad \Rightarrow b = 2a$

9.(B) $16\left(x^2 + 4x\right) - \left(y^2 - 4y\right) + 44 = 0$

$16(x+2)^2 - 64 - (y-2)^2 + 4 + 44 = 0$

$16(x+2)^2 - (y-2)^2 = 16$

$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$



$A = \int_{-2}^{\sqrt{6}} \left(2 - \left(x^2 - 4\right)\right) dx = \int_{-2}^{\sqrt{6}} (6 - x^2) dx = \left(6x - \frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$

$A = \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$

$A = \frac{12\sqrt{6}}{3} + \frac{28}{3}; \qquad A = 4\sqrt{6} + \frac{28}{3}$

10.(B) $\dfrac{dy}{dx} = \dfrac{y\phi'(x) - y^2}{\phi(x)} \Rightarrow \phi(x)dy = y\phi'(x)dx - y^2 dx$

$\Rightarrow \dfrac{y\phi'(x) - \phi(x)dy}{y^2} = dx$

$\Rightarrow \int d\left(\dfrac{\phi(x)}{y}\right) = \int dx \Rightarrow \dfrac{\phi(x)}{y} = x + c \Rightarrow y = \dfrac{\phi(x)}{x+c}$

11.(C) The parabola $\left(x + \dfrac{1}{2}\right)^2 + y^2 = \left(y + \dfrac{1}{y}\right)^2$

$\Rightarrow \quad y = x^2 + x \quad \Rightarrow f(x) = x^2 + x$ and $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \dfrac{\pi}{2}$

$\Rightarrow \quad f(x) = 0 \quad \Rightarrow x^2 + x = 0 \Rightarrow x = 0,\ x = -1$ two solution.

12.(C) Let $x + 1 = X \Rightarrow dx = dX$ & $y - 3 = Y \Rightarrow dy = dY$

$\dfrac{dY}{dX} = \dfrac{X^2 + Y}{X} \Rightarrow XdY - YdX = X^2 dX$

$\dfrac{XdY - YdX}{X^2} = dX \Rightarrow \int d\left(\dfrac{Y}{X}\right) = \int dX \Rightarrow \dfrac{Y}{X} = X + c$

$Y = X^2 + cX \Rightarrow y - 3 = (x+1)^2 + c(x+1)$ which passes through (1, 1)

$\Rightarrow -2 = 4 + 2c \Rightarrow c = -3 \Rightarrow y = x^2 - x + 1 = \left(x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4}$

So, $y_{\min} = \dfrac{3}{4}$

13.(B) $2\log_2 \log_2 x + \log_{1/2} \log_2\left(2\sqrt{2}x\right) = 1 \Rightarrow \log_2 (\log_2 x)^2 + \log_2\left(\dfrac{1}{\log_2 2\sqrt{2}x}\right) = 1$

$\Rightarrow \log_2\left[\dfrac{(\log_2 x)^2}{\log_2 2^{3/2} + \log_2 x}\right] = 1 \qquad \Rightarrow (\log_2 x)^2 = 2\left[\dfrac{3}{2} + \log_2 x\right]$

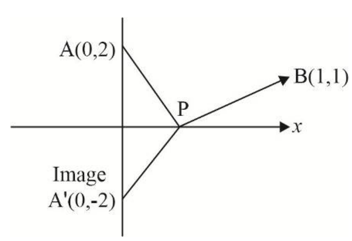
$\Rightarrow (\log_2 x)^2 - 2(\log_2 x) - 3 = 0 \qquad \Rightarrow \log_2 x = 3$ *or* $\log_2 x = -1$ (not possible)

$x = 2^3 \Rightarrow x = 8$ ($\because \log_2 x > 0$) $\qquad \Rightarrow x = 8$ only one solution

14.(C) Clearly $PA = PA'$

For least value of $PA' + PB$

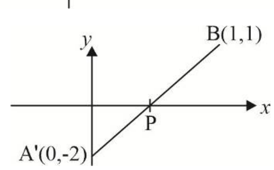
Points A', P, B are collinear



Equation of line $A'B$ is $y = 3x - 2$

Put $y = 0$ we get $P\left(\dfrac{2}{3}, 0\right)$

$\left(PA' + PB\right)_{\min} = A'B = \sqrt{10}$



15.(B) $\frac{1}{d}\left[\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+....+\frac{a_{4001}-a_{4000}}{a_{4000}a_{4001}}\right]=10,\ d=C\cdot D\cdot$

$$\frac{1}{d}\left[\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+....+\left(\frac{1}{a_{4000}}-\frac{1}{a_{4001}}\right)\right]=10$$

$$\frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_{4001}}\right]=10 \Rightarrow \frac{a_{4001}-a_1}{d\cdot a_1\cdot a_{4001}}=10$$

$$\frac{4000d}{d\cdot a_1\cdot a_{4001}}=10 \Rightarrow a_1a_{4001}=400,\ \text{also } a_2+a_{4000}=a_1+a_{4001}$$

$$|a_1-a_{4001}|=\sqrt{(a_1+a_{4001})^2-4a_1a_{4001}}=\sqrt{(50)^2-1600}=30$$

16.(D) $\left|\frac{1}{2}\begin{vmatrix}\alpha & \beta & 1\\ 5 & 6 & 1\\ 3 & 2 & 1\end{vmatrix}\right|=2 \Rightarrow 4\alpha-2\beta-8=\pm 4,$

$\Rightarrow 2\alpha-\beta=6$ *or* $2\alpha-\beta=2,$ therefore shortest length $=\frac{2}{\sqrt{5}}$

17.(D) $g(x)=f(-x)-f(x)=\frac{1}{1-e^x}-\frac{e^x}{e^x-1}=\frac{1+e^x}{1-e^x}=\frac{e^x-1+2}{1-e^x}=-1+\frac{2}{1-e^x}$

$$g'(x)=\frac{2e^x}{(1-e^x)^2}>0\ \forall x\in(0,1)$$

Clearly $g(x)$ is continuous and strictly increasing in (0, 1) hence $g(x)$ is one-one

18.(A) $f(x)$ is cont. and diff at $x = 0$

$$f'(x)=2x\cos\frac{1}{x}-\cos\frac{1}{x}$$

$\lim\limits_{x\to 0} f'(x)$ does not exist

So, $f'(x)$ is NOT continuous at $x = 0$

19.(D) $I=\pi^2\int_0^1\left(\cos\frac{\pi x}{2}\right)(x-[x])^0dx+\pi^2\int_1^2\left(\cos\frac{\pi x}{2}\right)(x-1)dx$

$$=\pi^2\left[\left[\frac{2}{\pi}\sin\frac{\pi x}{2}\right]_0^1+\left[(x-1)\left(\frac{2}{\pi}\sin\frac{\pi x}{2}\right)\right]_1^2-\int_1^2\frac{2}{\pi}\sin\frac{\pi x}{2}dx\right]$$

$$=\pi^2\left[\frac{2}{\pi}+0+\left(\frac{2}{\pi}\right)^2\left[\cos\frac{\pi x}{2}\right]_1^2\right]=\pi^2\left[\frac{2}{\pi}+0+\left(\frac{2}{\pi}\right)^2(-1)\right]=2\pi-4$$

20.(A) $\sum x=170,\ \sum x^2=2830$

Increase in $\sum x=10$, then $\sum x'=170+10=180$

Increase in $\sum x^2=900-400=500$, then $\sum x'=2830+500=3330$

Variance $=\frac{1}{n}\sum x'^2-\left(\frac{\sum x'}{n}\right)^2=\frac{3330}{15}-\left(\frac{180}{15}\right)^2=222-144=78$

SECTION-2

1.(5) Here $a^5b^3c^4 = (ab)^2(bc)^1(ca)^3$

Hence required coefficient is $\dfrac{6!}{2!1!3!}(1)^2\left(-\dfrac{2}{3}\right)^1\left(-\dfrac{1}{2}\right)^3$

2.(97) 2997

3.(108) Divisible by 6= divisible by 3 + even

Total ways = 48(0 is not included) + 60 (3 is not included) = 108

4.(3) $I = \displaystyle\int \frac{\frac{\cos x}{\sin^3 x}}{1+\cot^3 x}dx = \int \frac{\cot x\cdot \operatorname{cosec}^2 x}{(1+\cot x)(1+\cot^2 x-\cot x)}dx$, let $\cot x = t \Rightarrow -\operatorname{cosec}^2 x\ dx = dt$

$$I = -\int \frac{t}{(t+1)(t^2-t+1)}dt = -\frac{1}{3}\int\frac{dt}{1+t}+\frac{1}{6}\int\frac{2t-1}{t^2-t+1}dt-\frac{1}{2}\int\frac{dt}{t^2-t+1}$$

$$=\frac{-1}{3}\log_e|1+\cot x|+\frac{1}{6}\log_e|1-\cot x+\cot^2 x|-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cot x-1}{\sqrt{3}}\right)+c$$

$\alpha = \dfrac{-1}{3},\ \beta = \dfrac{1}{6},\ \gamma = -\dfrac{1}{\sqrt{3}}$. Therefore $18(\alpha+\beta+\gamma^2) = 18\left(\dfrac{-1}{3}+\dfrac{1}{6}+\dfrac{1}{3}\right) = 3$

5.(1) Coefficient of x^9 in $\left(\alpha x^2+\dfrac{1}{\beta x}\right)^{11} = {}^{11}C_r(\alpha x)^{11-r}\left(\dfrac{1}{\beta x}\right)^r$

$= {}^{11}V_r\alpha^{11-r}\cdot x^{33-3r}\cdot\beta^{-r}.\ x^{-r}$

Hence, $33-4r = 9$; $\quad -4r = -24$; $\quad r = 6$

Coefficient of x^9 is ${}^{11}C_6\alpha^5\beta^{-6}$ ….(i)

Coefficient of x^{-9} in $\left(\alpha x-\dfrac{1}{\beta x^3}\right)^{11}$

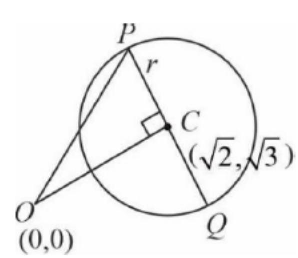
${}^{11}C_r(\alpha x)^{11-r}(-1)^r(\beta)^{-r}\left(x^{-3r}\right)$; $(-1)^r\,{}^{11}C_r\alpha^{11-r}x^{11-r-3r}\beta^{-r}$

$11-4r = -9 \quad\Rightarrow\quad r = 5$

Coefficient of x^{-9} is $= -{}^{11}C_5\alpha^6\beta^{-5}$

${}^{11}C_6\alpha^5\beta^{-6} = -{}^{11}C_5\alpha^6\beta^{-5} \quad\Rightarrow \alpha\beta = -1;\quad (\alpha\beta)^2 = 1$

6.(24) Area of $\Delta OCP = \dfrac{\sqrt{35}}{2} = \dfrac{1}{2}(OC)r = \dfrac{\sqrt{35}}{2}$



$\Rightarrow \quad \dfrac{1}{2}\sqrt{5}r = \dfrac{\sqrt{35}}{2} \Rightarrow r = \sqrt{7}$

Using parametric form of line with $C(x_1, y_1)$

$P\ \&\ Q \Rightarrow \left(x_1 \pm r\cos\theta,\ y_1 \pm r\sin\theta\right)$ … (i)

$m_{OC} = \dfrac{\sqrt{3}}{2} \quad \Rightarrow m_{PQ} = \dfrac{-\sqrt{2}}{\sqrt{3}} = \tan\theta \quad \Rightarrow \sin\theta = \dfrac{\sqrt{2}}{5}$

$\cos\theta = -\dfrac{\sqrt{3}}{\sqrt{5}}$; $\qquad x_1 = \sqrt{2},\ y_1 = \sqrt{3}$

(1) $\Rightarrow \left(\sqrt{2} \pm \sqrt{7}\left(\dfrac{-\sqrt{3}}{\sqrt{5}}\right), \sqrt{3} \pm \sqrt{7}\left(\dfrac{\sqrt{2}}{\sqrt{5}}\right)\right)$

$P(a_1, b_1) \pm P\left(\sqrt{2} - \dfrac{\sqrt{21}}{\sqrt{5}},\ \sqrt{3} + \dfrac{\sqrt{14}}{\sqrt{5}}\right)$

$Q(a_2, b_2) \pm Q\left(\sqrt{2} + \dfrac{\sqrt{21}}{\sqrt{5}},\ \sqrt{3} - \dfrac{\sqrt{14}}{\sqrt{5}}\right)$

$a_1^2 + a_2^2 + b_2^2 = 24$

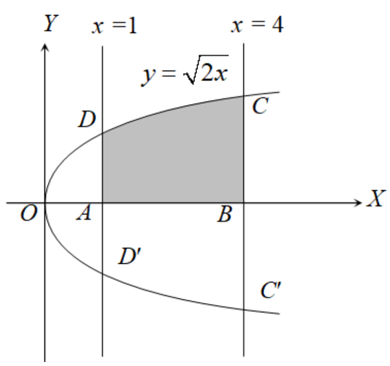
7.(8) Here $AB = A \,\&\, BA = B \Rightarrow A^2 = A \Rightarrow \begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$.

$a^2 = a, b(a+d-1) = 0,\ d^2 = d$

a	b	d
0	0	0
0	1	–1, 0, 1
1	0	–1, 0, 1
1	1	0

Total 8 matrices

8.(28) Required area $= CDD'C'' = 2 \times ABCD$



$= 2\displaystyle\int_1^4 \sqrt{2}x^{1/2}dx = \dfrac{28\sqrt{2}}{3}$ sq. unit.

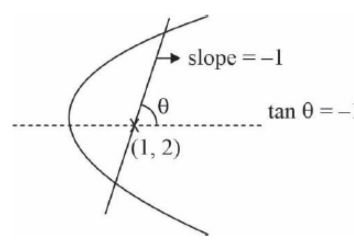
9.(16) $y^2 - 4y = 8x + 4$; $\qquad (y-2)^2 = 8(x+1)$

$a = 2$, vertex (–1, 2), focus (1, 2)

Since x-int of focal chord is 3

$\therefore$ It passes through (3, 0)

Equation of focal chord is $y - 0 = -1(x-3)$; $\quad x + y = 3$



Length of focal chord $= 4a\,\text{cosec}^2\,\theta \quad = 4 \times 2 \times \left[1 + (-1)^2\right] = 16$

10.(3) Let $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c,\ f'(-1) = 3a - 2b + c = 0$

$f''(x) = 6ax + 2b,\ f''(1) = 0 = 6a + 2b$

$\therefore\ b = -3a \qquad \ldots \text{(i)}$

$c = -9a \qquad \ldots \text{(ii)}$

$f(-1) = 10,\ f(1) = -6$

$-a + b - c + d = 10 \qquad \text{... (iii)}$

$a + b + c + d = -6 \qquad \ldots \text{(iv)}$

From (i), (ii), (iii) and (iv) ; $a = 1,\ b = -3,\ c = -9,\ d = 5$

$f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$

Hence local minima exists at $x = 3$